Exercise 13

(a) What quantities are given in the problem?

(b) What is the unknown?

(c) Draw a picture of the situation for any time t.

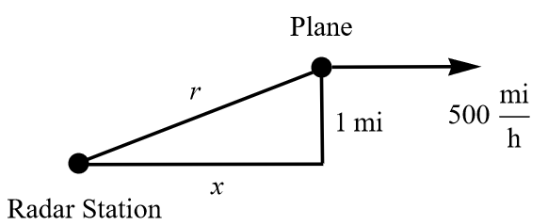
(d) Write an equation that relates the quantities.

(e) Finish solving the problem.

A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

Solution

The plane's speed (dx/dt) and height above the ground (1 mi) are given. The rate that r, the distance from the station to the plane, is increasing is unknown.



The relationship between r and x is given by the Pythagorean theorem.

$$r^2 = x^2 + 1^2 \quad \rightarrow \quad \begin{cases} r = \sqrt{x^2+1} \\ x = \sqrt{r^2-1} \end{cases}$$

The rate that r is increasing is the derivative of r with respect to time t.

$$\begin{aligned} \frac{dr}{dt} &= \frac{1}{2}(x^2+1)^{-1/2} \cdot \frac{d}{dt}(x^2+1) \\ &= \frac{1}{2}(x^2+1)^{-1/2} \cdot \left(2x \cdot \frac{dx}{dt}\right) \\ &= \frac{x}{\sqrt{x^2+1}} \frac{dx}{dt} \\ &= \frac{\sqrt{r^2-1}}{r} \frac{dx}{dt} \end{aligned}$$

Therefore, the rate that r is increasing when $r = 2$ is

$$\left.\frac{dr}{dt}\right|_{r=2} = \frac{\sqrt{2^2-1}}{2}\left(500\ \frac{\text{mi}}{\text{h}}\right) = 250\sqrt{3}\ \frac{\text{mi}}{\text{h}}.$$